Magic Madgwick Filter for Attitude Estimation

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***Abstract*—This project presents different approaches for attitude tracking based on IMU data. Two simple methods and a Madgwick filter are implemented and compared. The Madgwick filter is able to combine the benefits of both simple methods to improve attitude estimation.**

I. Introduction/Problem Statement

The goal of this project was to implement three different methods to estimate attitude based on IMU data. First, two simple methods to calculate the orientation based off of the gyroscope and accelerometer seperately are to be developed. Lastly, a Madgwick filter is implemented to estimate the attitude of the IMU.

II. Estimating Attitude from Accelerometer

The accelerometer measures the accelerations in three directions. These accelerations can be used to calculate the orientation of the accelerometer at any given instant.

The IMU body axes are defined in Figure 1.

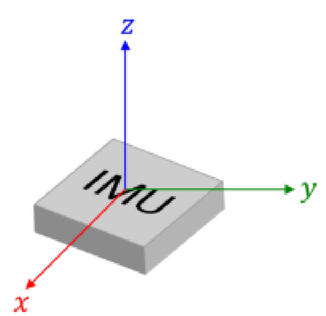

Fig. 1. IMU Axes [1]

Using gravity, the roll (ϕ) and pitch (θ) can be calculated using the following equations.

$$\phi = tan^{-1}\left(\frac{a_y}{\sqrt{{a_x}^2 + {a_z}^2}}\right) \tag{1}$$

$$\theta = tan^{-1}\left(\frac{-a_x}{\sqrt{{a_y}^2 + {a_z}^2}}\right) \tag{2}$$

If the accelerometer is completely horizontal, then yaw (ψ) is impossible to detect (with just the IMU). However, if the accelerometer is not horizontal, then the following equation can be used for yaw.

$$\psi = tan^{-1}\left(\frac{\sqrt{{a_y}^2 + {a_x}^2}}{a_z}\right) \tag{3}$$

However, the accelerometer data given must first be converted to physical units. This is done using the following equation where a'_x is the raw data given and a_x is the acceleration in meters per second squared.

$$a_x = \frac{a'_x + b_{a,x}}{s_x} \tag{4}$$

In this equation, the bias and scale factor (b and s) are given and were obtained by using a least squares regression between the accelerometer and Vicon data. A similar conversion is done for the accelerations in both the y and z directions.

After the conversion is done, Eqs 1, 2, and 3 are used to determine the roll, pitch, and yaw respectively.

The results for this method are found in Section V.

III. Estimating Attitude from Gyroscope

The IMU also has a gyroscope, which measures the angular velocities. The simplest method for obtaining attitude from a gyroscope is integrating the angular velocities to find the orientation at any point.

The IMU axes are the same as shown in Figure 1.

Since integration will be used to calculate the attitude, an initial condition (for each of the three angles) is needed. An average of the initial 200 points (roughly 2 seconds) of the Vicon data is used for the initial conditions for the training sets (where the Vicon data is available). For the test set, the gyroscope is initially assumed to be at rest (all angles are zero).

Similar to the accelerometer, the gyroscope data is not given in physical units, and must undergo a conversion to get to radians per second.

$$\omega = \frac{3300}{1023} * \frac{\pi}{180} * .3 * (\omega' - b_g) \tag{5}$$

The bias factor b is calculated as the average of the first 200 samples of the gyroscope (this assumes that the gyroscope is at rest).

In this equation ω' is the raw data that is obtained directly from the IMU.

After determining the values of the angular velocity in physical units, the next step is to integrate the angular velocity over each time step to obtain the position at each time step. However, the measured gyroscope rates are in the body axis frame, while the desired angles are euler angles. Therefore, at every time step, the measured gyro rate must be converted using the following equation:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}_i = \begin{bmatrix} 1 & sin(\phi)tan(\theta) & cos(\phi)tan(\theta) \\ 0 & cos(\phi) & -sin(\phi) \\ 1 & sin(\phi)sec(\theta) & cos(\phi)sec(\theta) \end{bmatrix}_i \begin{bmatrix} \dot{\omega_x} \\ \dot{\omega_y} \\ \dot{\omega_z} \end{bmatrix}_i \quad (6)$$

This step is done at every time set using the Euler angles from the current time step to solve for the euler angle rate at the current time step. Then this rate is used with the current Euler angles to solve for the Euler angles at the next time step.

$$\begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix}_{i+1} = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix}_i + \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}_i dt \quad (7)$$

The results for this method are found in Section V.

IV. Madgwick Filter

Using just the accelerometer and gyroscope seperately to calculate attitude provides two less than ideal estimates. Using the accelerometer leads to very little drift, however the high frequency behavior is not very well captured. The gyroscope is able to capture some of this high frequency behavior, but because of the constant integration, can drift over time. the Madgwick filter looks to combine both of these methods to provide a better attitude estimate [2].

A. Gyroscope Model

The angular rate of the gyroscope is measured in the body axes. Using the quaternion for the previous orientation and the current gyroscope data, the rate in the earth frame can be obtained.

$$^S_E\dot{q}_{\omega,t} = \frac{1}{2} {^S_E}\hat{q}_{est,t-1} \otimes {^S}\,\omega \quad (8)$$

B. Accelerometer Model

Unlike above, simple equations will not be used to solve for the euler angles from the accelerations. This is because the method used above actually has infinite angle combinations (orientations about the gravity axis).

Instead, an optimization function problem is created. The orientation of the accelerometer that gets the measured gravitational field in the sensor frame to align with a predefined reference direction of the field in the earth frame is the goal. Basically, if we know what gravity is in the earth frame, and what the gravity field is in the sensor frame, then we can calculate the orientation of the sensor in the earth frame.

$$^S_E q_{\Delta,t} = {^S_E}\hat{q}_{est,t-1} - \mu_t \frac{\Delta f}{||\Delta f||} \quad (9)$$

In this equation, μ_t is the step size of the gradient descent.

C. Data Combination

When we look to combine the data from the gyroscope and the accelerometer, we can solve for an optimum μ_t for the accelerometer to avoid overshooting (non-physical overshooting).

After obtaining this optimum step size, we plug this back into the equation, and after certain approximations, we end up with an equation for the quarternion rate.

$$^S_E\dot{q}_{est,t} = \frac{1}{2} {^S_E}\hat{q}_{\omega,t} - \beta {^S_E}\dot{\hat{q}}_{\epsilon,t} \quad (10)$$

where,

$$^S_E\dot{\hat{q}}_{\epsilon,t} = \frac{\Delta f}{||\Delta f||} \quad (11)$$

is the direction of the error vs. time.

After calculating the quaternion rate, the integration is performed as in the gyroscope method above.

What this can be thought of is, similar to above, the gyroscope is being used to solve for the rate of change of the attitude. However, now the accelerometer, which gives the attitude (not rate), is being used to help ”ground” the measurement during the constant integration process. This comes in the form of the direction of error vs. time term. This should help keep the accuracy at higher frequency motion, while not letting the measurement drift too much.

Another modification which can be added to the madgwick filter is a scaling factor for the accelerometer and gyroscope components. This is similar to what is done in the complementary method. Instead of having the .5 multiplier for both, the following can be done.

$$^S_E\dot{q}_{est,t} = \alpha {^S_E}\hat{q}_{\omega,t} - (1-\alpha) \beta {^S_E}\dot{\hat{q}}_{\epsilon,t} \quad (12)$$

where α is between 0 and 1. This variable could be tuned depending on the expected behavior.

V. Results

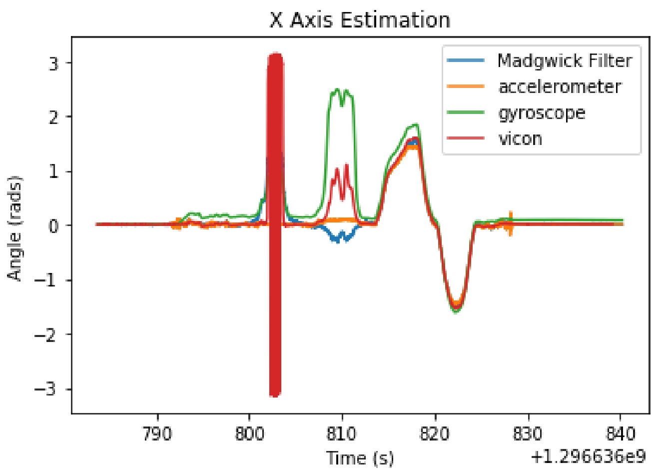

Fig. 2. Train 1 Roll

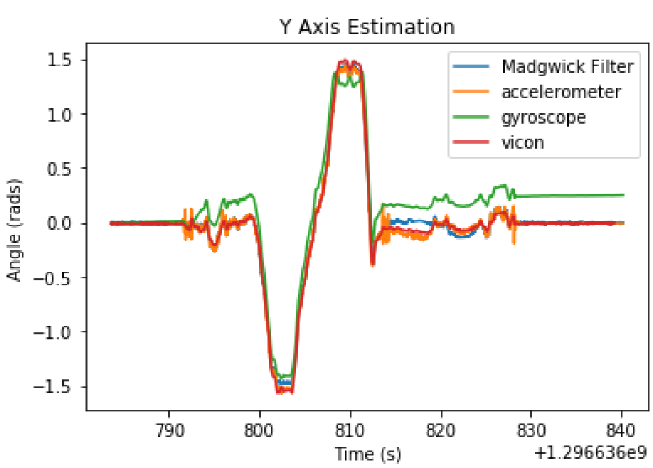

Fig. 3. Train 1 Pitch

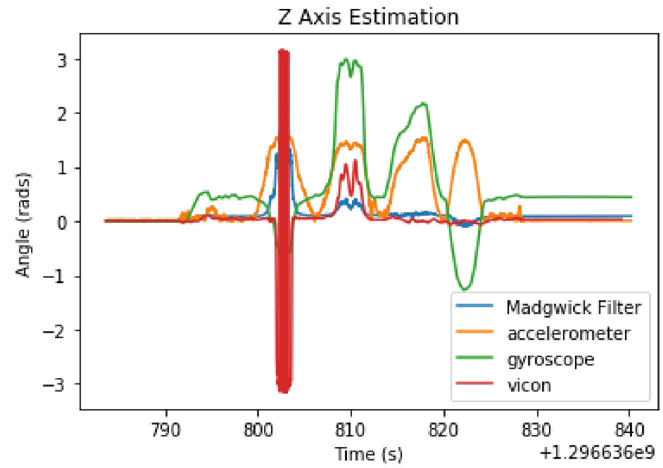

Fig. 4. Train 1 Yaw

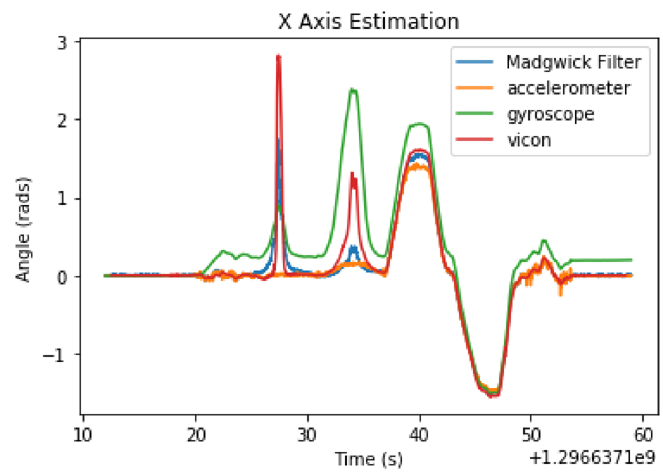

Fig. 5. Train 2 Roll

Fig. 6. Train 2 Pitch

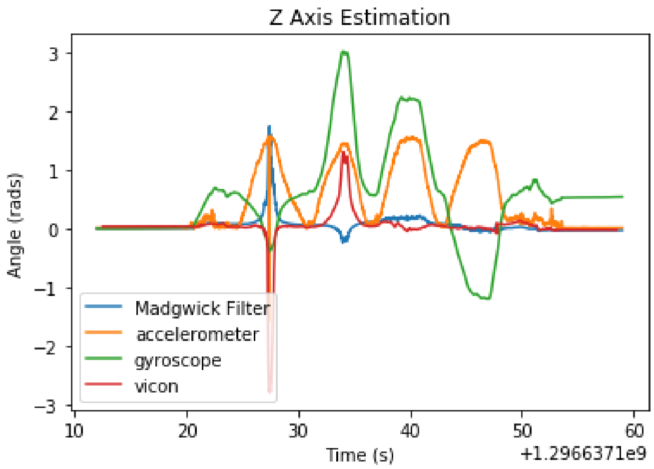

Fig. 7. Train 2 Yaw

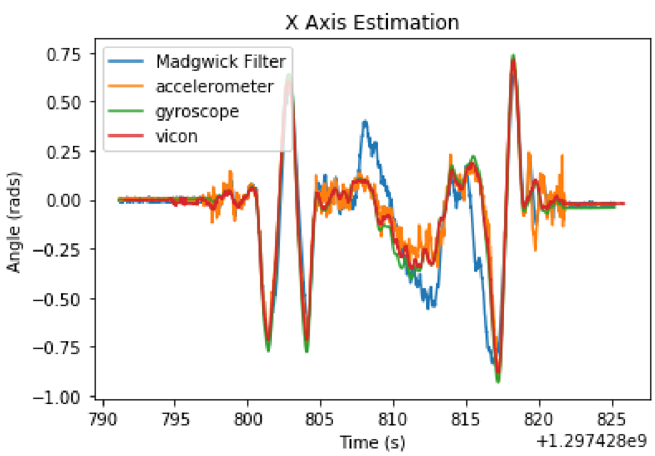

Fig. 8. Train 3 Roll

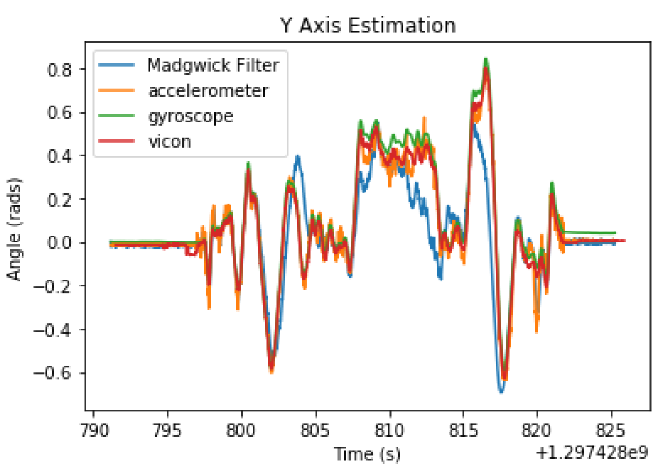

Fig. 9. Train 3 Pitch

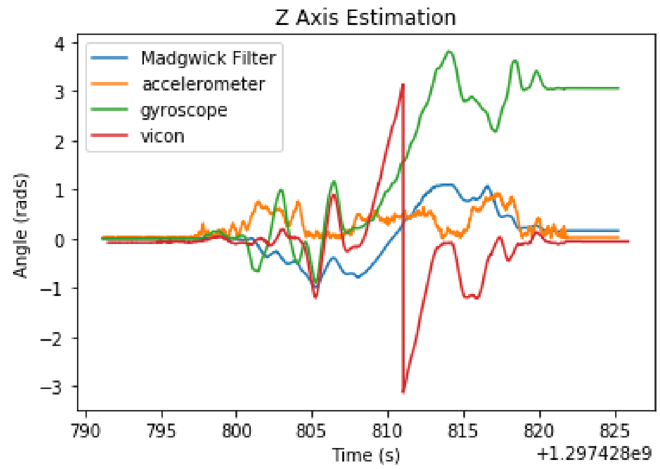

Fig. 10. Train 3 Yaw

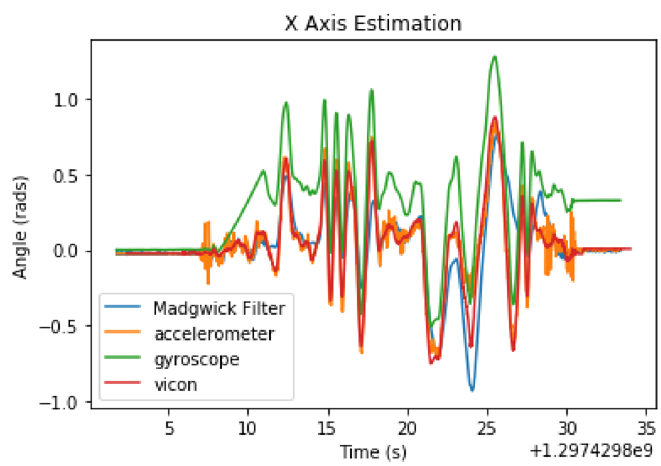

Fig. 11. Train 4 Roll

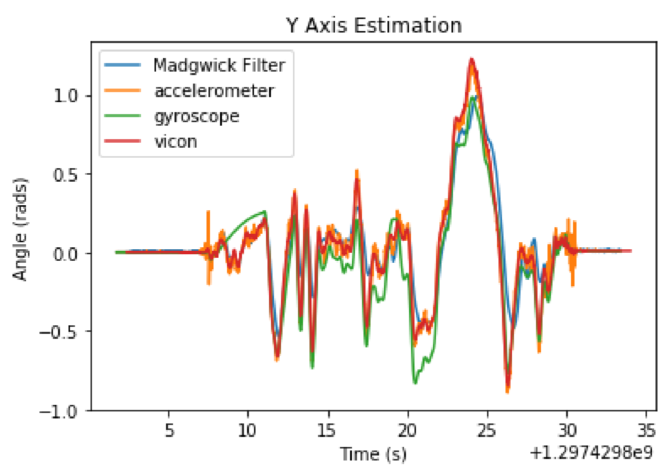

Fig. 12. Train 4 Pitch

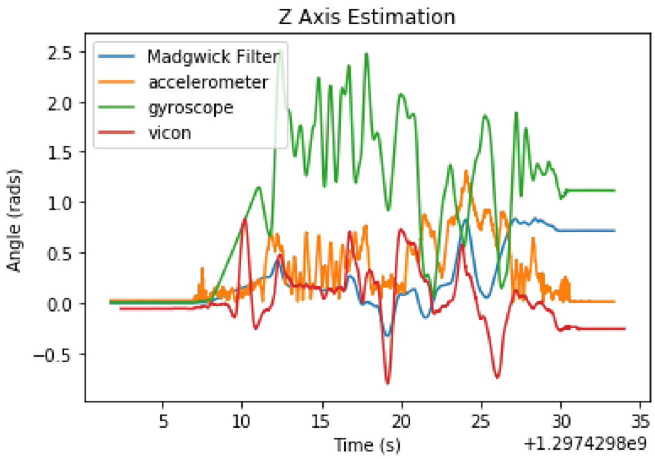

Fig. 13. Train 4 Yaw

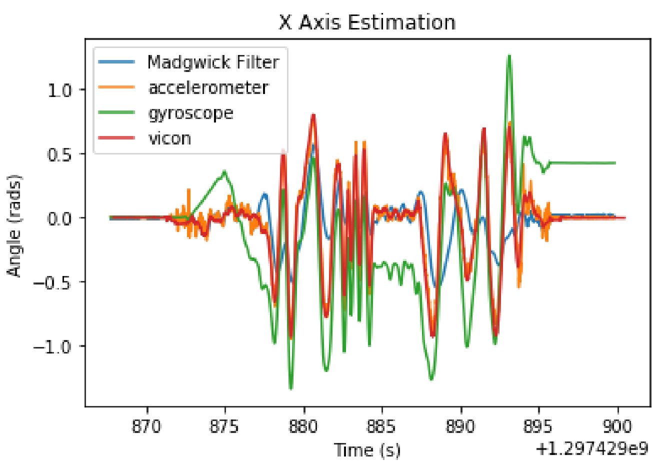

Fig. 14. Train 5 Roll

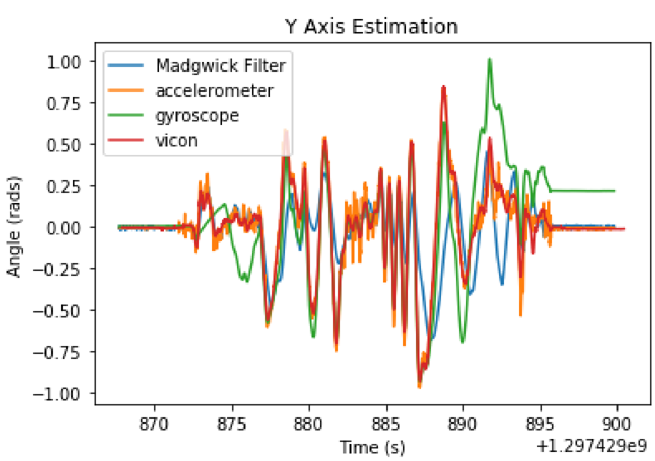

Fig. 15. Train 5 Pitch

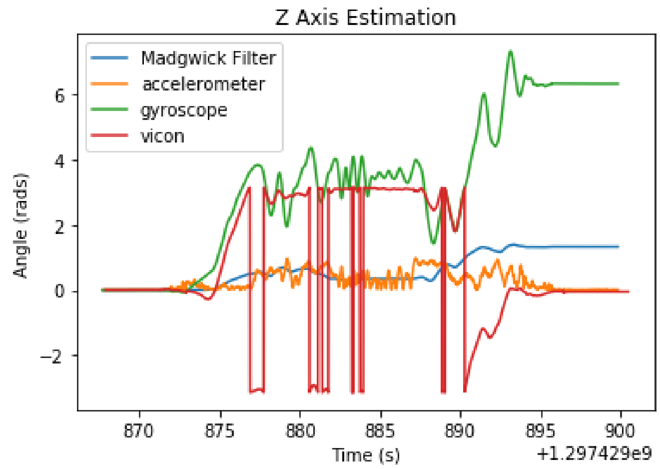

Fig. 16. Train 5 Yaw

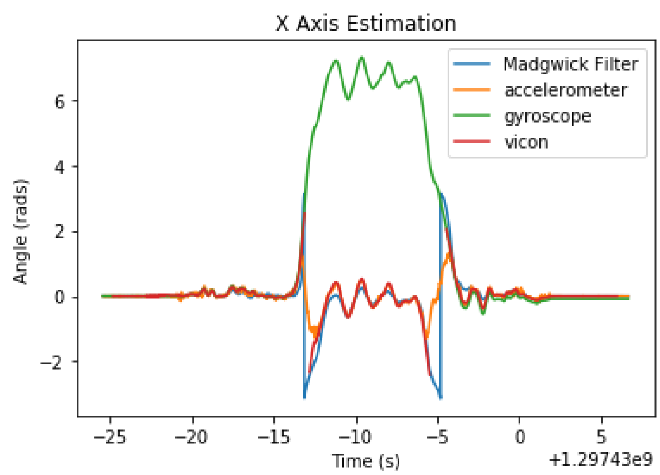

Fig. 17. Train 6 Roll

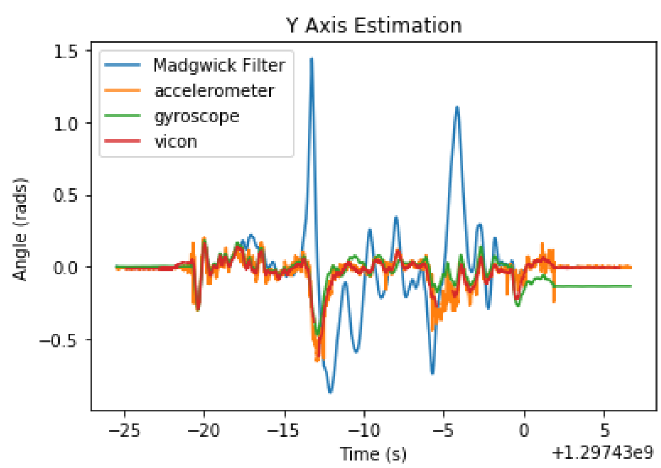

Fig. 18. Train 6 Pitch

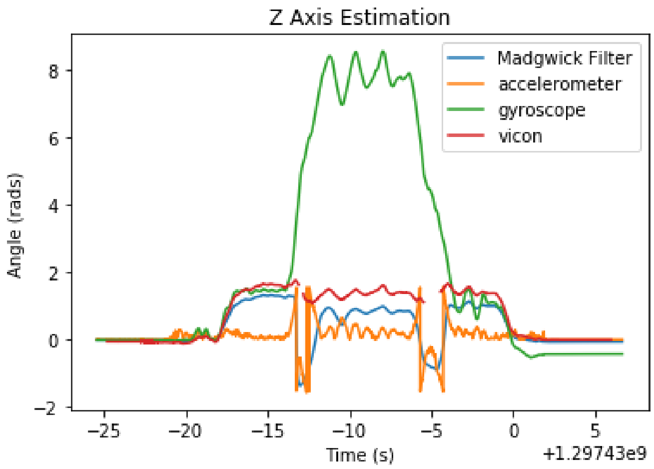

Fig. 19. Train 6 Yaw

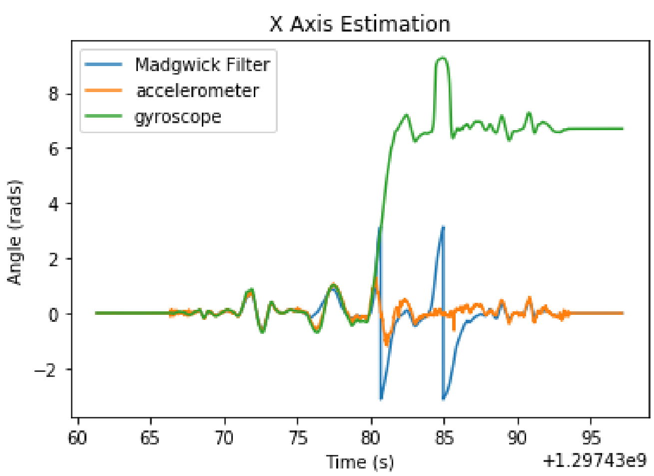

Fig. 20. Test 7 Roll

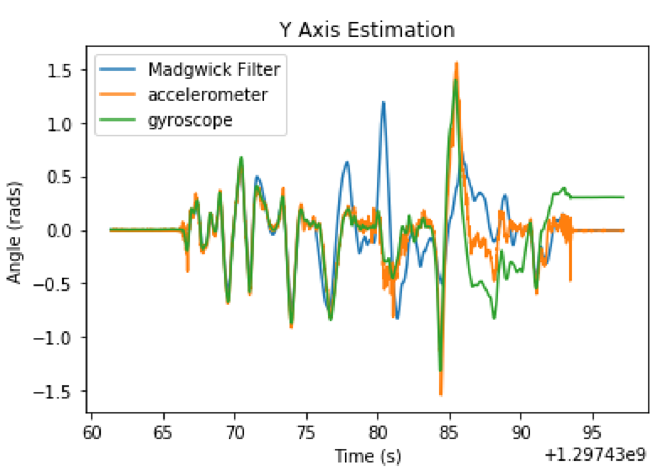

Fig. 21. Test 7 Pitch

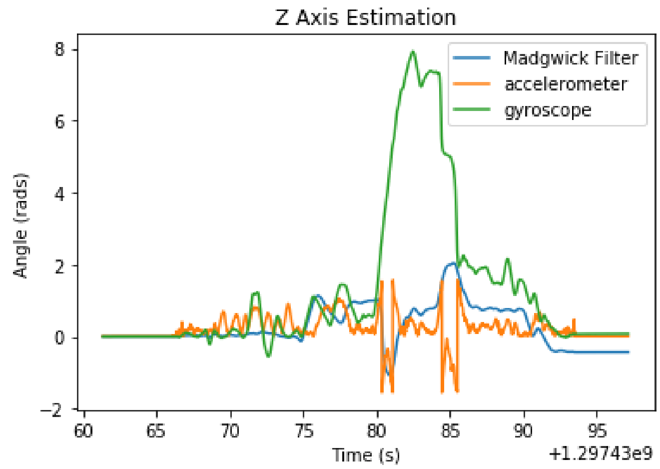

Fig. 22. Test 7 Yaw

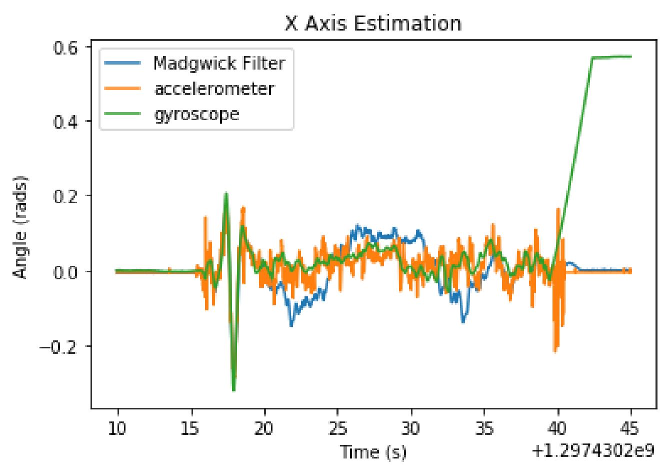

Fig. 23. Test 8 Roll

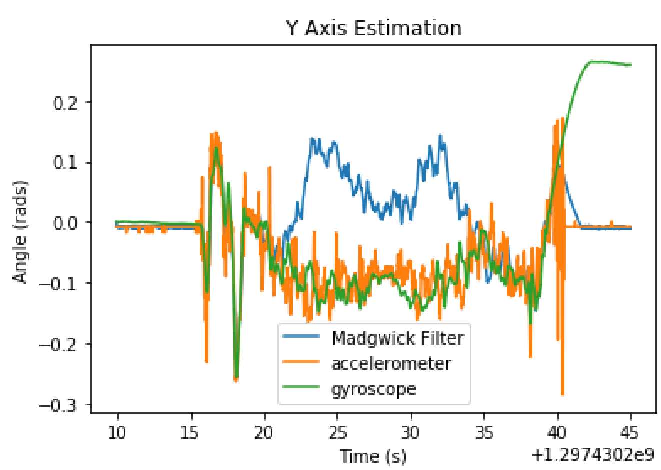

Fig. 24. Test 8 Pitch

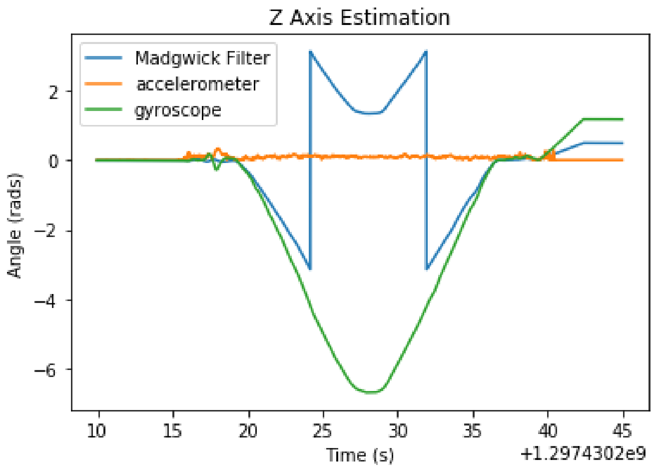

Fig. 25. Test 8 Yaw

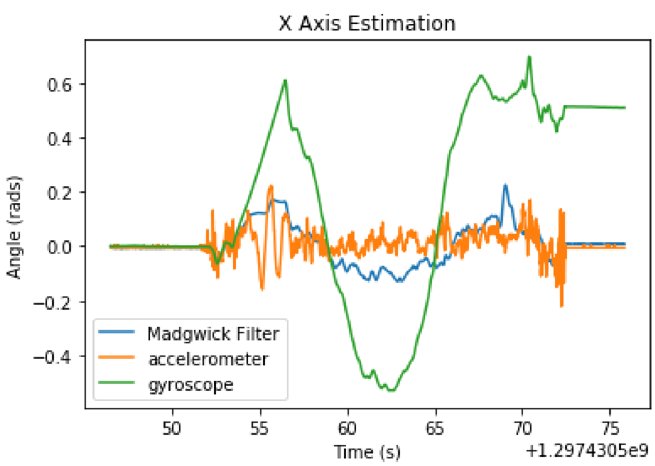

Fig. 26. Test 9 Roll

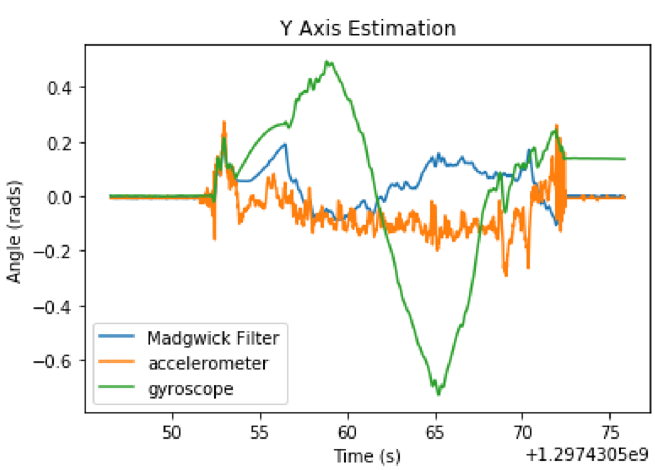

Fig. 27. Test 9 Pitch

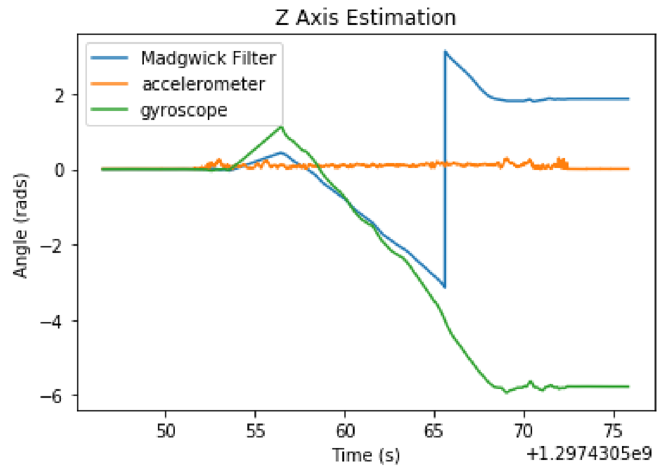

Fig. 28. Test 9 Yaw

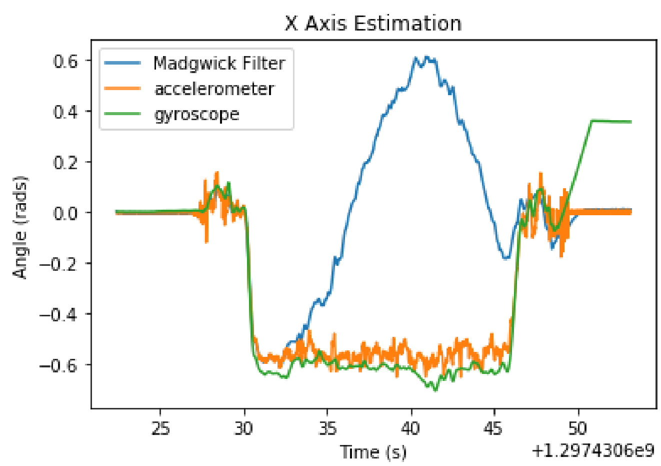

Fig. 29. Test 10 Roll

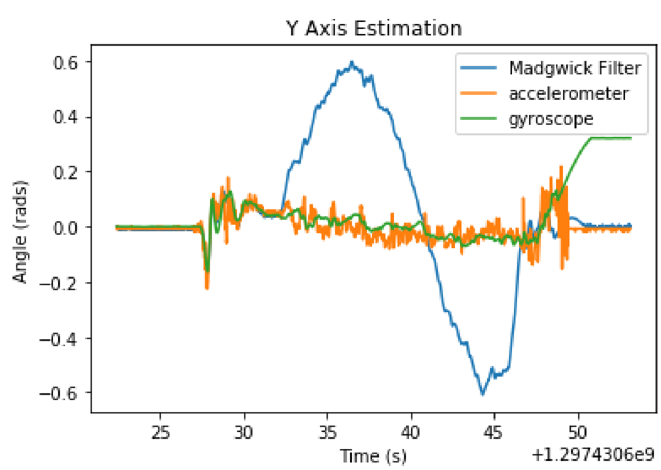

Fig. 30. Test 10 Pitch

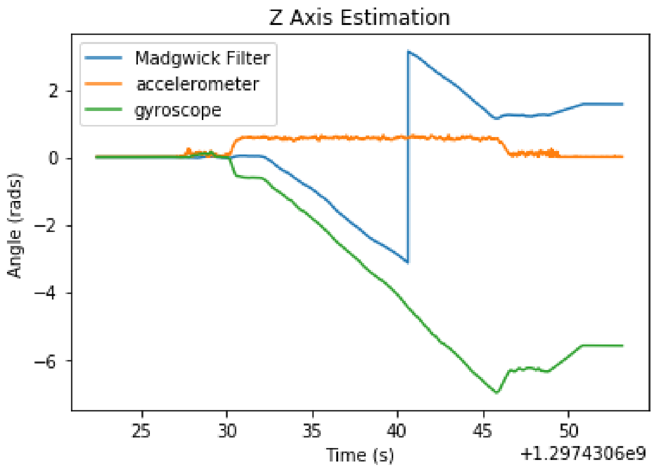

Fig. 31. Test 10 Yaw

VI. Conclusion

Looking at the results, we are able to see several overall trends. the accelerometer is normally decently accurate for roll and pitch, but struggles for yaw (as expected). It has very little drift compared to the gyroscope, which is especially noticable at the end of the tests, after several minutes of testing.

Tuning the α and β values for the Madgwick filters was done for the training sets to best match the Vicon truth data. Then, the behavior of the gyroscope and accelerometer for the test sets (without Vicon truth data) was compared to those from the training data. Based on this, α and β values were chosen for the test data.

The rotplot videos can be found on Youtube and the links are in the README.md file.

References

[1] ENAE788 Class 2 Part 2 Slides
[2] Sebastian OH Madgwick, Andrew JL Harrison, and Ravi Vaidyanathan. ”Estimation of IMU and MARG orientation using a gradient descent algorithm.” 2011 IEEE international conference on rehabilitation robotics. IEEE, 2011.
[3] Some Code taken from learnopencv.com/rotation-matrix-to-euler-angles/