Non-stinky Unscented Kalman Filter for Attitude Estimation

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***Abstract*—In this report the Unscented Kalman Filter is implemented and tested with an IMU dataset that was provided for ENAE788M: Hands on Autonomous Aerial Robotics. Results from the Unscented Kalman Filter are compared with a tuned Madgwick Filter, both of which estimate attitude using a combination of Accelerometer and Gyroscope data. Vicon data is used as the ground truth to compare the estimated attitudes and reveal the advantages of the Unscented Kalman Filter.**

I. Introduction

In many aerospace and robotics applications, accurately measuring the system's orientation plays a vital role for supplying feedback to the autopilot or the controllers. A very popular sensor for measuring orientation is an IMU (Inertial Measurement unit). It consists of tri-axis gyroscopes and accelerometers. Gyroscopes measures the angular velocity, which can be integrated over time from some known initial condition to estimate the sensor's orientation. The numerical integration technique is prone to accumulation of errors over time which leads to a drift in the estimated orientation from the true orientation. On the other hand, an accelerometer measures the earth's gravitational field and therefore can provide orientation estimates from an absolute frame of reference. However, any translational motion will result in corrupted measurements of earth's gravity and consequently the orientation estimates suffers. This particular problem can be handled by using an orientation filter that estimates a single estimate of orientation by fusing the accelerometer and gyroscope measurements provided by an IMU. The Unscented Kalman Filter is a nonlinear filter that does this through the use of a Guassian probability distribution approximated by Sigma Points that predicts an orientation that is compared to the measurement orientation. This method allows for an attitude estimate to quickly converge and is advantageous over an Extended Kalman filter, which linearizes the model and can be computationally costly.
Link to the result videos: Click Here

II. Mathematical Details

A. IMU Data Pre-Processing

Data gathered from the IMU must first be pre-processed to convert into physical units and to negate bias inherent to the instrumentation. The following expression describes the conversion of raw accelerometer readings $\boldsymbol{a} = [a_x \ a_y \ a_z]^T$ to acceleration data in m/s^2.

$$\tilde{a_i} = 9.81(a_i * s_i + b_{a,i}) \tag{1}$$

where s_i is a scale factor of the accelerometer for each axis and $b_{a,i}$ is the bias for each axis.

To convert the raw gyro angular velocity reading $\boldsymbol{\omega} = [\omega_x \ \omega_y \ \omega_z]^T$ into angular velocity data in rad/s, the following expression is used.

$$\tilde{\omega_i} = \frac{3300}{1023} * \frac{\pi}{180} * .3 * (\omega_i - b_{g,i}) \tag{2}$$

where $b_{g,i}$ is calculated as the average of the first 200 raw angular velocity reading samples for each axis of rotation.

B. UKF

A detailed description with mathematical derivation of the Unscented Kalman Filter can be found in [1]. The state vector and measurements vector at any time t is given by the following vector expressions.

$$\begin{aligned} \hat{x}_t &= [\hat{\phi}_t \ \hat{\theta}_t \ \hat{\psi}_t \ \hat{\boldsymbol{\omega}}_t^T]^T \\ z_t &= [\boldsymbol{a}_t \ \boldsymbol{\omega}_t]^T \end{aligned} \tag{3}$$

The steps for implementing the UKF are described below.

1) Prediction Step: At time step t to perform the unscented transform on the previous state (represented as a Gaussian with mean μ_{t-1} and covariance Σ_{t-1}) we need to generate Sigma Points, which are calculated as follows.

$$\begin{aligned} \chi_{t-1}^{[0]} &= \mu_{t-1} \\ \chi_{t-1}^{[i]} &= \mu_{t-1} + \left(\sqrt{(n+\lambda)\Sigma_{t-1}}\right)_i \text{ for } i = 1, ..., n \\ \chi_{t-1}^{[i]} &= \mu_{t-1} - \left(\sqrt{(n+\lambda)\Sigma_{t-1}}\right)_i \text{ for } i = n+1, ..., 2n \end{aligned}$$

These Sigma points are now propagated through the Process model which essentially captures the kinematics and dynamics of the states to be estimated. This step is described below.

$$\bar{\chi}_t^* = f(\chi_{t-1}, u_t) \tag{4}$$

where

$$f(\hat{x}_{t-1}, u_t) = \begin{bmatrix} I_{3\times3} & A(\hat{x}_{t-1})\Delta t \\ 0_{3\times3} & I_{3\times3} \end{bmatrix} \hat{x}_{t-1}$$

$$A(\hat{x}_{t-1}) = \begin{bmatrix} 1 & \sin(\hat{\phi}_{t-1})\tan(\hat{\theta}_{t-1}) & \cos(\hat{\phi}_{t-1})\tan(\hat{\theta}_{t-1}) \\ 0 & \cos(\hat{\phi}_{t-1}) & -sin(\hat{\phi}_{t-1}) \\ 0 & \sin(\hat{\phi}_{t-1})\sec(\hat{\theta}_{t-1}) & \cos(\hat{\phi}_{t-1})\sec(\hat{\theta}_{t-1}) \end{bmatrix} \tag{5}$$

Now. the mean and covariance of the transformed sigma points is calculated as follows

$$\begin{aligned} \bar{\mu}_t &= \sum_{i=0}^{2n} w_m^{[i]} \bar{\chi}_t^{*[i]} \\ \bar{\Sigma}_t &= \sum_{i=0}^{2n} w_c^{[i]} \left(\bar{\chi}_t^{*[i]} - \bar{\mu}_t\right) \left(\bar{\chi}_t^{*[i]} - \bar{\mu}_t\right)^T + Q_t \end{aligned} \tag{6}$$

where Q_t is the prediction/process model's noise covariance matrix.

2) Correction Step: The updated mean and covariance from the prediction step is used to generate a new set of Sigma points before passing them through the measurement model.

$$\begin{aligned} \chi_t^{[0]} &= \bar{\mu}_t \\ \chi_t^{[i]} &= \bar{\mu}_t + \left(\sqrt{(n+\lambda)\bar{\Sigma}_t}\right)_i \quad \text{for } i = 1, ..., n \\ \chi_t^{[i]} &= \bar{\mu}_t - \left(\sqrt{(n+\lambda)\bar{\Sigma}_t}\right)_i \quad \text{for } i = n+1, ..., 2n \end{aligned}$$

The Sigma points are propagated through the measurement model to get a prediction of the measurement sigma points. This is described by the following expression.

$$\bar{Z}_t^* = h(\chi_t) \tag{7}$$

where

$$h(\hat{x}_t) = \begin{bmatrix} -9.81\sin(\hat{\theta}_t) \\ 9.81\cos(\hat{\theta}_t)\sin(\hat{\phi}_t) \\ 9.81\cos(\hat{\phi}_t)\cos(\hat{\theta}_t) \\ \hat{\boldsymbol{\omega}}_t \end{bmatrix} \tag{8}$$

Now, a mean and covariance of the predicted measurement sigma points is calculated as follows,

$$\begin{aligned} \hat{z}_t &= \sum_{i=0}^{2n} w_m^{[i]} \bar{Z}_t^{*[i]} \\ S_t &= \sum_{i=0}^{2n} w_c^{[i]} \left(\bar{Z}_t^{*[i]} - \hat{z}_t\right) \left(\bar{Z}_t^{*[i]} - \hat{z}_t\right)^T + R_t \end{aligned} \tag{9}$$

where R_t is the measurment model's noise covariance. The Kalman gain can now be computed as follows.

$$\begin{aligned} \bar{\Sigma}_t^{\chi,Z} &= \sum_{i=0}^{2n} w_c^{[i]} \left(\bar{\chi}_t^{*[i]} - \bar{\mu}_t\right) \left(\bar{Z}_t^{*[i]} - \hat{z}_t\right)^T \\ K_t &= \bar{\Sigma}_t^{\chi,Z} S_t^{-1} \end{aligned} \tag{10}$$

The mean and covariance calculated in the prediction step is corrected using the measurement data z_t obtained from the sensors.

$$\begin{aligned} \mu_t &= \bar{\mu}_t + K_t\left(z_t - \hat{z}_t\right) \\ \Sigma_t &= \bar{\Sigma}_t - K_t S_t K_t^T \end{aligned} \tag{11}$$

III. Results

Datasets 1 to 6 include Vicon measurements treated as ground truth to compare the performance of the Unscented Kalman Filter and the Madgwick Filter. The Datasets 7 to 10 are the evaluation/test datasets and do not contain the Vicon measurements.

A. Dataset 1

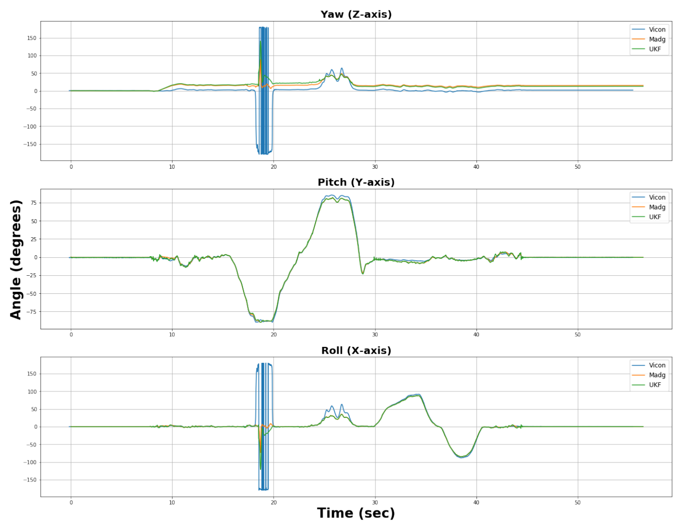

Fig. 1. Comparision of Attitude Estimation using the various filters with the Vicon Ground Truth for the first datatset.

B. Dataset 2

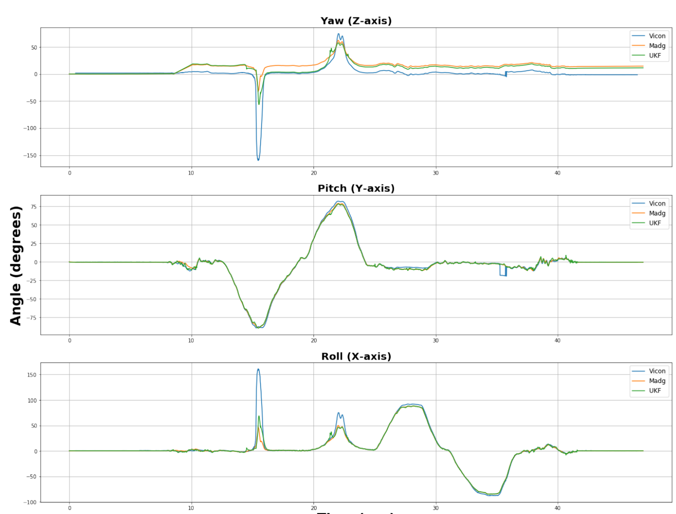

Fig. 2. Comparision of Attitude Estimation using the various filters with the Vicon Ground Truth for the second datatset.

C. Dataset 3

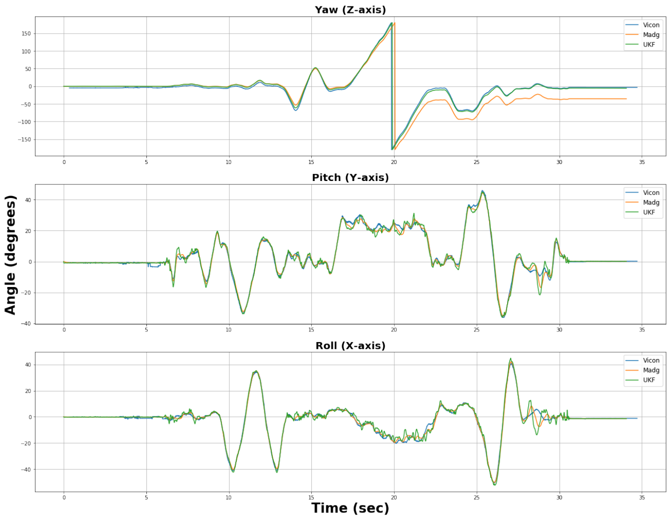

Fig. 3. Comparision of Attitude Estimation using the various filters with the Vicon Ground Truth for the third datatset.

D. Dataset 4

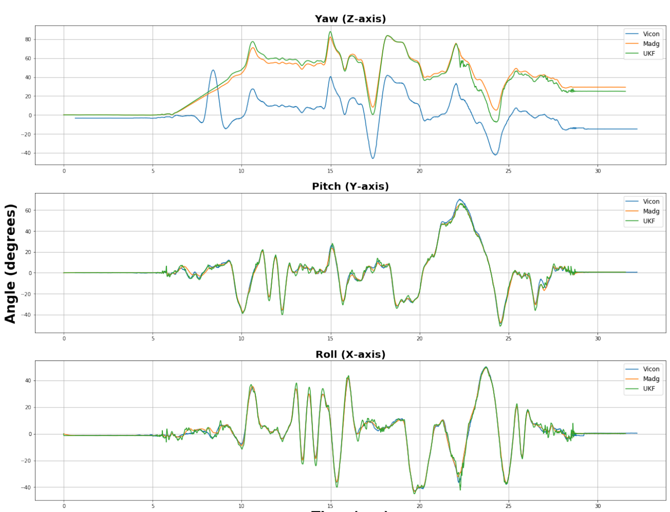

Fig. 4. Comparision of Attitude Estimation using the various filters with the Vicon Ground Truth for the fourth datatset.

E. Dataset 5

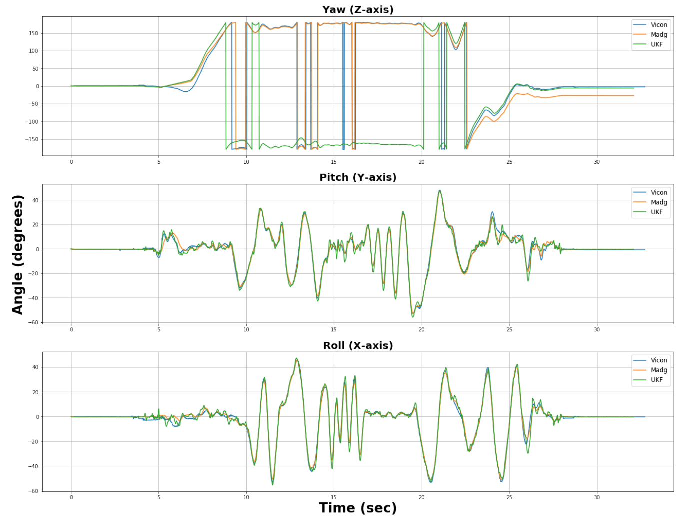

Fig. 5. Comparision of Attitude Estimation using the various filters with the Vicon Ground Truth for the fifth datatset.

F. Dataset 6

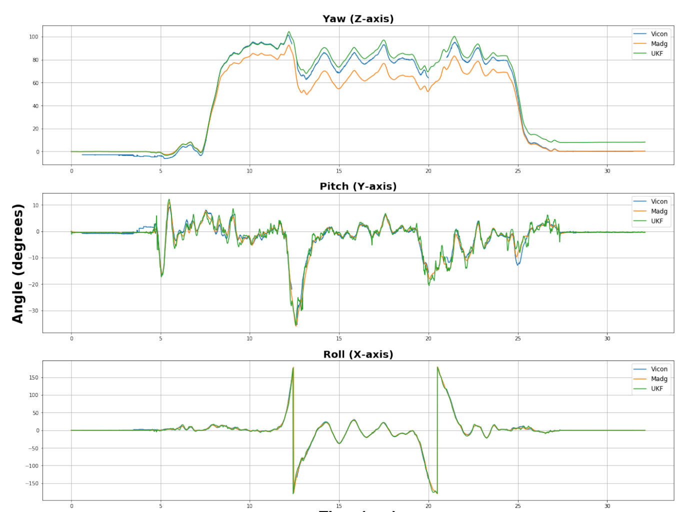

Fig. 6. Comparision of Attitude Estimation using the various filters with the Vicon Ground Truth for the sixth datatset.

G. Dataset 7

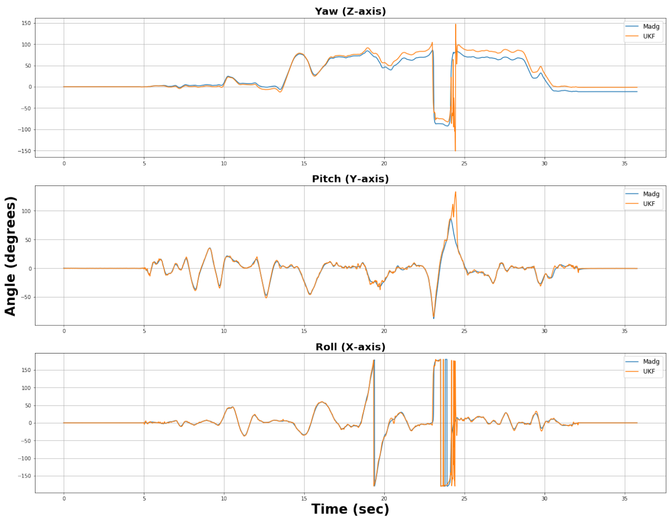

Fig. 7. Comparision of Attitude Estimation using the various filters for the seventh datatset.

H. Dataset 8

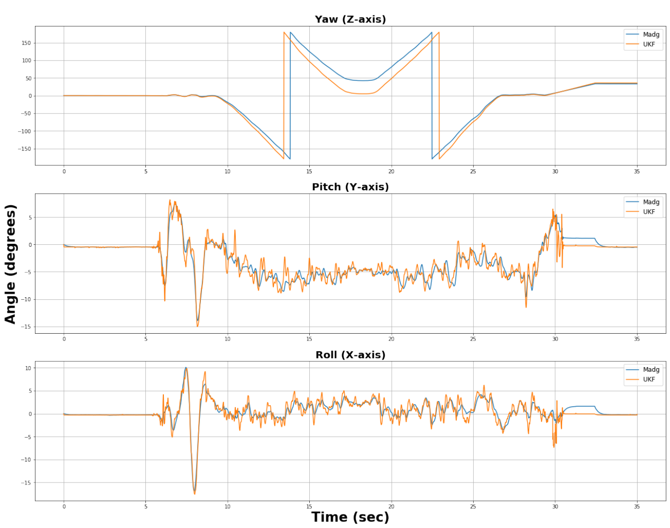

Fig. 8. Comparision of Attitude Estimation using the various filters for the eighth datatset.

I. Dataset 9

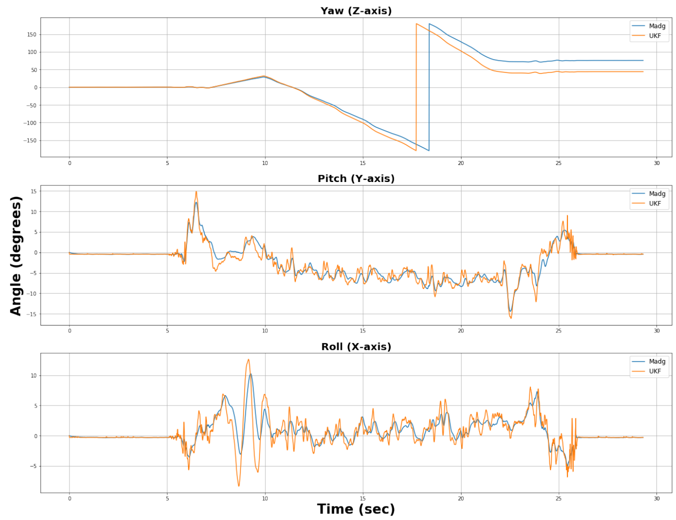

Fig. 9. Comparision of Attitude Estimation using the various filters for the ninth datatset.

J. Dataset 10

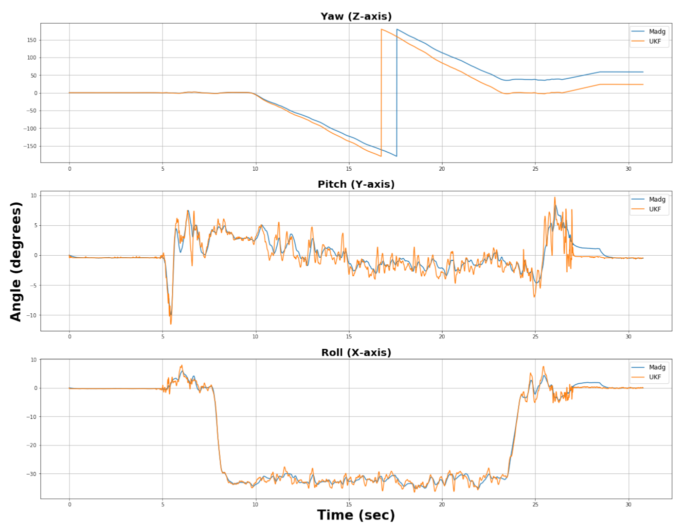

Fig. 10. Comparision of Attitude Estimation using the various filters for the tenth datatset.

IV. Conclusion

The results show that both the Madgwick Filter and Unscented Kalman Filter are very good at estimating roll and pitch attitude. However, yaw estimates suffer slightly due to the accelerometer's inherent inability to measure accelerations about the yaw axis. As a result, yaw estimates tend to drift for both filters due to drift in the gyro measurement and inaccurate corrections in UKF with the accelerometer's data. In multiple cases however, the performance of the Unscented Kalman Filter in estimating yaw orientation surpassed that of the Madgwick Filter, proving it to be a superior alternative to the linear Madgwick Filter.

REFERENCES

[1] Simon J. Julier and Jeffrey K. Uhlmann, *A New Extension of the Kalman Filter to Nonlinear Systems*